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A DISCUSSION OF: "RANDOM FATIGUE FAILURE OF A MULTIPLE LOAD PATH REDUNDANT STRUCTURE"

by

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where k_{cr} is the critical stress intensity factor.

The growth rate law becomes

$$\frac{da}{dn} = \frac{(\Delta\sigma)^4}{M_4} a^2,$$

where $\Delta\sigma$ is the variation of the cyclicly applied stress.

Integration, subject to the condition $a = a_o$ when $n = o$, gives

$$\frac{1}{a_o} - \frac{1}{a} = \frac{(\Delta\sigma)^4}{M_4} n,$$

which becomes, in terms of the ultimate load,

$$\sigma_{cr} = (\sigma_{cr_o}{}^2 - \frac{k_{cr}^2 (\Delta\sigma)^4 n}{M_4})^{1/2}$$

The preceding formula will not necessarily reproduce experimental data because of the following simplifications employed: the crack growth law is approximate, the critical stress intensity factor is actually dependent on crack length for small a, and finite boundaries of the cracked body are not taken into account. However, the implications are clear. Rational procedures for estimating the reduction in ultimate strength of a member due to fatigue loadings can be obtained from information on crack propagation rates and fracture mechanics failure criteria coupled with solutions to relevant elastic boundary value problems for cracked bodies.

A second point of question is the use of the Miner equation for life estimates under variable amplitude fatigue loadings.

Interpreting fatigue in terms of crack propagation, it seems clear that the damaging effect of a given load level in terms of the increment of growth caused at a crack tip will depend not only on the number of times a given load is applied, but also on the prior load levels which may have served to blunt or sharpen the crack and on the length of the crack when a particular load level is applied. Still, present experimental data on crack propagation can suggest a reasonable alternative to the Miner equation only in some special cases of block loading of constant amplitude stress cycles.

A few results will now be given which indicate a method of approach to the statistical description of fatigue based on crack propagation and fracture mechanics failure criteria. Consider a through the thickness crack of length a in a plane body. Assume the body is loaded with arbitrarily distributed boundary tractions all of which are proportional to some parameter $s = s(t)$ which varies in a random fashion with time.

The stress intensity factor for such a loading, coming from a linear elastic boundary value problem, is assumed known and can always be expressed in the form

$$k(t) = s(t)\, f(a)$$

As $s(t)$ varies with time, the crack length a will increase, affecting the concentration of stress experienced at the crack tip through the function $f(a)$. Finally, a maximum in $s(t)$ will occur at a sufficiently high level such that the crack will propagate in an unstable manner due to an excess of the critical stress intensity factor, k_{cr}, and the member will fail. To express this process in probabilistic terms, let

$p(n)$ = Probability that the stress intensity factor k_n in the nth load peak of $s(t)$ will be sufficiently great to cause failure, and let

$F(n)$ = Probability that the crack will have survived the first n load peaks without an unstable propagation.

It can be shown, through well-known techniques for first occurence time problems in the theory of probability, that

$$F(n) = \exp \left\{ - \int_0^n p(n)dn \right\},$$

provided that $p(n)$ is extremely small compared to one and that $p(n)$ has a negligible correlation with $p(n-1)$, $p(n-2)$, etc. Here, n is regarded as a continuous rather than a discrete variable.

An expression for $p(n)$ which considers the statistical variation in load levels and which considers the critical stress intensity factor as a random material variable is

$$p(n) = \int_0^\infty g(x) \text{ Prob}\left\{k_n > x\right\} dx$$

where

$g(x)dx$ = Probability that $x < k_{cr} < x+dx$

and

$\text{Prob}\left\{k_n > x\right\}$ = Probability that the stress intensity factor $k_n > x$.

The result for p(n) is most conveniently expressed in terms of the crack length a, and the notation $p(a) = p(n)$ will be used, where it is recognized that a is a function of n dependent on the crack growth rate law. F(n) is also expressible in terms of crack length. Suppose a crack has the initial length a_o. Then F(n) is the probability that the cracked body will survive n load peaks, or viewing a as a function of n, it is the probability $F(a|a_o)$ that a crack will grow to a length a without unstable propagation given that its initial length was a_o. Thus $F(a|a_o) = F(n)$. Using this new notation, one has

$$F(a|a_o) = \exp\left\{-\int_{a_o}^{a} p(a)\frac{1}{G(a)}da\right\},$$

where $G(a) = \frac{da}{dn}$ is the rate of crack propagation or average crack extension per load peak.

Work to date on crack propagation has not led to a satisfactory expression for evaluating G(a), but it is known that this quantity is dependent on the stress intensity factor variation felt at a crack tip. Thus, for a given material and random load parameter s(t), one can experimentally determine G(a) in the form of a polynomial

$$G(a) = \frac{da}{dn} = \sum_{r=1}^{N} g_r \left\{f(a)\right\}^r$$

where, the constants, g_r, will depend on material properties and statistical constants of the random process s(t). Once determined for some particular crack configuration (that is, some particular f(a)), the result will be valid for any arbitrary crack configuration since it is known the crack growth rate in cracked bodies loaded in different ways is the same if the stress intensity factor felt at the crack tip is the same. It should also be mentioned that elementary work on the plasticity problem for a cracked body indicates that only even powers of f(a) should be taken in the preceding series.

Explicit expressions could be given for p(a), and also on the basis of rather scant experimental results for G(a), for $F(a|a_o)$. These will be deferred for publication in a report which the authors are presently preparing under a N.A.S.A. research grant at Lehigh University.

Before closing, some remarks will be made on applications of the statistical approach to fatigue given herein. In aircraft operation, cracks are often detected by pre-flight inspection, and an important problem is the estimation of the residual life of the cracked body as it is subjected to the type of random loading exerted on the structure by atmospheric turbulence. Usually, estimates of remaining life are extremely sensitive to the initial crack length and slight errors in measurement can effect the predicted fatigue life greatly. Thus, the survival probability F(a) can be estimated from an integral of the form

$$F(a) = \int_0^\infty F(a|a_o)h(a_o)da,$$

where $h(a_o)$ is a probability density for the initial crack length a_o based on the measured value but with a sufficient variance to account for possible measurement errors.

Another application is the problem of the joining of two cracks and prediction of such quantities as the expected life before joining and expected remaining life after joining. More difficult applications come in life predictions for multi-cracked redundant structures where load redistribution due to crack propagation must be taken into account.

Reference

(1) Paris, Paul C., The Fracture Mechanics Approach To Fatigue, 1963, Sagamore Conference.